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THEORY OF TWO-PHASE TRANSPIRATION COOLING SYSTEMS. II

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UDC 536.248.2

A statistical mode of bubbling in porous solids is formulated. The theory developed is used for closing the system of transport equations for a two-phase transpiration cooling system.

We shall consider here the problem of hydrodynamics and heat exchange in two-phase transpiration cooling systems. In our previous paper [1], we considered the case of a porous solid consisting of capillaries with equal or variable cross sections. However, although this model is of practical interest [2], the more often encountered porous materials with a highly complicated void structure remain outside the scope of applicability of the developed theory. We shall consider here the model of liquid bubbling in porous systems of the fluidized-solid and use it as a basis for writing the macroscopic continuum equations which describe the hydrodynamics and heat exchange in two-phase transpiration cooling systems.

Statistical Model of Bubbling in a Porous Solid

The bubbling of a liquid filtering through a porous material occurs in the following manner. A vapor bubble which has formed in a certain pore grows in volume, fills the entire pore, and then passes into the neighboring pores along the links connecting this pore with the others. As a result, a vapor-filled cavity forms inside

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 4, pp. 663-668, October, 1976. Original article submitted October 28, 1974.

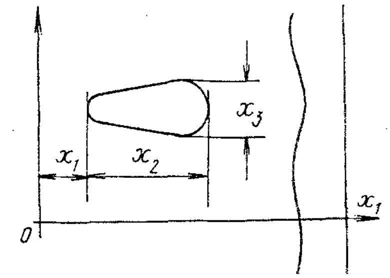

Fig. 1. Coordinates of a vapor cavity in the phase space.

the porous material. This cavity moves inside the porous material together with the liquid flow. As it moves, the cavity continues to grow because it is fed by evaporation from the liquid.

The liquid column in front of the vapor cavity moves faster than the liquid behind which there is no vapor cavity. This occurs because of the fast growth of the cavity volume. As a result of acceleration of the liquid column, the pressure drop across it increases, which hinders the development of another cavity in this column. Thus, the development of two cavities side by side is unlikely, as is the case in the development of a vapor lock in an individual capillary [3].

Let us write the equation describing the statistical behavior of such vapor cavities. Each cavity is characterized by the following set of parameters (see Fig. 1): x_1 is the cavity coordinate directed across the porous specimen layer; x_2 is the cavity length; and x_3 is the transverse dimension of the cavity. In this phase space, we introduce the density of vapor cavities $N(t, x_1, x_2, x_3)$ and define it so that, for the unit cross section of the specimen, $N(t, x_1, x_2, x_3)dx_1dx_2dx_3$ is equal to the number of cavities in the phase space cell $dx_1dx_2dx_3$ near the point x_1, x_2, x_3. We write the transport equation for this quantity:

$$\frac{\partial N}{\partial t}+\sum_{i=1}^{3}\frac{\partial v_i N}{\partial x_i}=R^+. \tag{1}$$

The source R^+ is defined by

$$R^+ = A(x_1)\delta(x_2)\delta(x_3), \tag{2}$$

where the δ-function accounts for the fact that the dimensions of the nascent cavities are of the same order as the dimensions of the porous structure cell, i.e., they are close to zero. If we separate in $A(x_1)$ the term depending explicitly on the coolant superheat and, through it, also on x_1, as was done in [1], we can rewrite (2) in the following form:

$$R^+ = ks_l\,\omega(x_1)\delta(x_2)\delta(x_3), \tag{3}$$

where k depends on the porous structure characteristics, for instance, the porosity $\varepsilon(x_1)$. Therefore, generally, $k = k(x_1)$. The expression for $\omega(x_1)$ is given in [1]. The quantity s_l is defined below.

Equation (1) holds if either the density of cavities is low or the layer within which the cavities evolve is sufficiently thin, so that mutual interaction of cavities can be neglected.

The liquid–vapor interface inside the specimen has a highly irregular shape, which varies constantly with a fluctuation amplitude comparable to the thickness of the liquid bubbling region inside the specimen. The shape and the position of this surface can vary under the influence of several factors: the translational motion of the liquid forced through the specimen; the fact that the liquid column pushed forward by the growing vapor cavity moves faster than the liquid without a cavity behind it; and the interaction between the cavity and the interface at the moment when the cavity reaches it and collapses. Only a statistical description of interface makes sense. Subsequently, we shall assume that any straight line parallel to the x_1 axis intersects the liquid–vapor interface only once. Let us construct a cylinder with the transverse cross section s, whose generatrix is along the x_1 axis. This cylinder cuts off a part of the liquid–vapor interface. Different elements $d\sigma$ of this surface can be located at different points along the x_1 axis. We shall denote by $\sigma_{x_1}|_{x_1=x}$ the total projection on the x_1 axis of such elements, located at the point $x_1 = x$. We now introduce the quantity $\sigma_{x_1}|_{x_1=x}/s$. The mean of this quantity with respect to a large number of cylinders $\xi(x, t)$ represents the probability that a part of the liquid–vapor interface will be found at the point $x_1 = x$. It follows from this definition that

$$\int_{-\infty}^{+\infty}\xi(x,t)dx = 1. \tag{4}$$

The quantity $\xi(x, t)$ can be considered not only as the probability, but also as the average share of the interface projection per unit cross section of the specimen at the point x. Taking this into account, we can write the transport equation for $\xi(x, t)$.

The flux of this quantity is the product between it and the rate at which it intersects the unit area. In the case under consideration, a part of the liquid–vapor interface, ξ_1, intersects the area at the rate v_1^*, while the other part, ξ_2, intersects it at the rate v_2^*. Here v_1^* is the velocity of the interface when it is not preceded by a vapor cavity, and v_2^* is the velocity of the interface when the latter is preceded by a vapor cavity. The value of v_2^* depends on the cavity parameters, $v_2^* = v_2^*(x_1, x_2, x_3)$, and, moreover, $\xi_1 + \xi_2 = \xi$.

Let us introduce the quantity ξ_2', the density of ξ_2, defining it as the part of ξ_2 which is preceded by cavities with the parameters x_1, x_2, x_3:

$$\xi_2(x) = \int_{-\infty}^{x} \int_{0}^{\infty} \int_{0}^{x-x_1} \xi_2' dx_2 dx_3 dx_1. \tag{5}$$

The flux of $\xi(x, t)$ is then written

$$r_\xi = v_1^* \xi_1 + \int_{-\infty}^{x} \int_{0}^{x-x_1} \int_{0}^{\infty} v_2^* \xi_2' dx_3 dx_2 dx_1. \tag{6}$$

From the definition of ξ_2', it follows that

$$\xi_2' = \pi x_3^2 N(x_1, x_2, x_3, t). \tag{7}$$

As a result, we write r_ξ in the following form by taking into account (6):

$$r_\xi = v_1^* \xi + \int_{-\infty}^{x} \int_{0}^{x-x_1} \int_{0}^{\infty} (v_2^* - v_1^*) \pi x_3^2 N(x_1, x_2, x_3, t) dx_1 dx_2 dx_3 \tag{8}$$

and the following transport equation holds for $\xi(x, t)$:

$$\frac{\partial \xi}{\partial t} + \frac{\partial v_1^* \xi}{\partial x} = -\frac{\partial}{\partial x} \int_{-\infty}^{x} \int_{0}^{x-x_1} \int_{0}^{\infty} (v_2^* - v_1^*) \pi x_3^2 N(x_1, x_2, x_3, t) dx_3 dx_2 dx_1 + R^+ + R^-, \tag{9}$$

where R^+ and R^- are the terms reflecting the interaction between the interface and the vapor cavity at the moment when the latter collapses. We can readily write explicit expressions for the collision integrals R^+ and R^-. For this, we introduce the quantity $\Phi(x_1, x_2, x_3)$, which is equal to the flux of vapor cavities with the parameters x_2 and x_3, the origin of which intersects the unit area which is located at the point x_1 and moves at the velocity v_2^*. Using $\Phi(x_1, x_2, x_3)$, we represent the collision integrals R^+ and R^- in the following form:

$$R^+ = \int_{0}^{\infty} \int_{0}^{\infty} \pi x_3^2 \Phi(x_1, x_2, x_3) \xi(x_1 + x_2) dx_3 dx_2, \tag{10}$$

$$R^- = -\int_{0}^{\infty} \int_{0}^{\infty} \pi x_3^2 \Phi(x_1 - x_2, x_2, x_3) \xi(x_1) dx_3 dx_2. \tag{11}$$

The quantity $\Phi(x_1, x_2, x_3)$ is expressed in terms of $N(x_1, x_2, x_3, t)$ as follows:

$$\Phi(x_1, x_2, x_3) = (v_1 + v_2 - v_2^*) N(x_1, x_2, x_3, t). \tag{12}$$

Equations (1) and (9) provide a statistical description of the evolution of vapor cavities and of the liquid–vapor interface during coolant bubbling in a porous solid.

Transport Equation Based on the Statistical Model of Coolant Bubbling in a Porous Solid.

In order to complete the statement of the problem, it is necessary to determine the velocities v_1, v_2, v_3, v_1^*, and v_2^* in Eqs. (1) and (9) and write the macroscopic transport equations on the basis of the described statistical model of bubbling.

Since $\eta_v < \eta_l$, the vapor cavity moves faster than the surrounding liquid for the same pressure distribution. In a layer whose thickness is of the order of the cavity radius the liquid in front of the cavity is pushed forward at a rate equal to the velocity of the front boundary of the cavity. Further on along the coolant flow, the effect of the cavity is less pronounced, and the liquid there filters through at the rate v_l. If, in the momentum balance, we take into account the fact that a part of the liquid filters through at a rate equal to the vapor velocity, the model becomes unnecessarily complicated. Therefore, we shall subsequently consider that all of the liquid filters through at the mean velocity $\bar{v}_l$, while vapor has the mean velocity $\bar{v}_\mathrm{v}$. We thereby assume that $v_1^* = v_2^*$.

The velocity of the liquid–vapor interface is expressed in terms of the liquid velocity in pores v_l in the following manner:

$$v_1^* = v_l - \frac{\tau}{\rho_l}, \tag{13}$$

where τ is the evaporation rate, which, according to [4], is given by

$$\tau = \frac{\varkappa\alpha\sqrt{\mu}}{\sqrt{2\pi R^* T}}\,[p_s(T) - p]. \tag{14}$$

The rate at which the vapor cavity moves is equal to the vapor velocity in pores v_v. If the cavity expands equally in the direction of motion and in the opposite direction, we have

$$v_1 = v_\mathrm{v} - \frac{v_2}{2}. \tag{15}$$

The velocities v_2 and v_3 characterize the growth of the longitudinal and transverse dimensions of the vapor cavity. These values can be estimated by considering the vapor mass balance in the cavity. It can be shown that, in the limiting case of very intensive phase transition, when the cavity grows primarily on account of the vapor entering it, the equation of vapor mass balance in the cavity is written as follows:

$$\rho_\mathrm{v}\frac{d\Omega}{dt} = \tau s, \tag{16}$$

where Ω is the volume of the cavity, and s is its surface area. Assuming that the longitudinal and the transverse dimensions of the cavity increase equally, i.e., $v_2 = v_3$, and considering (14), we obtain the following estimate:

$$v_2 \sim \varkappa\alpha\sqrt{\frac{R^*T}{2\pi\mu}}\cdot\frac{p_s(T) - p}{p}\cdot\frac{2(x_2 + x_3)}{2x_2 + x_3}, \tag{17}$$

where x_2 and x_3 are the longitudinal and the transverse dimensions of the cavity.

We introduce the quantity s_v, equal to the fraction of the unit cross section of the specimen where vapor filters through. It is expressed in the following manner in terms of $N(x_1. x_2, x_3, t)$ and $\xi(x, t)$:

$$s_\mathrm{v} = \int\limits_{-\infty}^{x}\int\limits_{x-x_1}^{\infty}\int\limits_{0}^{\infty}\pi x_3^2 N(x_1, x_2, x_3, t)\,dx_3\,dx_2\,dx_1 + \int\limits_{-\infty}^{x}\xi(x_1)\,dx_1. \tag{18}$$

The similar quantity s_l for the liquid is then equal to

$$s_l = 1 - s_\mathrm{v}. \tag{19}$$

By using these quantities, we can express the flux density of the coolant mass, the momentum flux density, the energy flux density, and the other quantities figuring in transport equations.

Let us write the final system of transport equations for the case of one-dimensional steady-state filtration of a vapor–liquid mixture through a porous solid. This system includes the equation of mass conservation for the coolant,

$$\frac{d}{dx}\,[\varepsilon_\mathrm{sur} s_\mathrm{v}\rho_\mathrm{v} v_\mathrm{v} + \varepsilon_\mathrm{sur} s_l\,\rho_l v_l] = 0, \tag{20}$$

the equation of motion of the vapor–liquid mixture,

$$\varepsilon_\mathrm{sur}\frac{dp}{dx} = -\frac{d}{dx}(\varepsilon_\mathrm{sur} s_\mathrm{v}\rho_\mathrm{v} v_\mathrm{v}^2 + \varepsilon_\mathrm{sur} s_l\,\rho_l\,v_l^2) - \frac{\varepsilon_\mathrm{sur}}{k}(\eta_\mathrm{v} v_\mathrm{v} s_\mathrm{v} + \eta_l\,v_l\,s_l), \tag{21}$$

the equation of motion of the vapor phase,

$$\varepsilon_\mathrm{sur} s_\mathrm{v}\frac{dp}{dx} = -\frac{d}{dx}(\varepsilon_\mathrm{sur} s_\mathrm{v}\rho_\mathrm{v} v_\mathrm{v}^2) - \frac{d}{dx}(\varepsilon_\mathrm{sur} s_l\,\rho_l\,v_l)v_l - \frac{\varepsilon_\mathrm{sur}}{k}\eta_\mathrm{v} v_\mathrm{v} s_\mathrm{v}, \tag{22}$$

and the equation of energy transport,

$$\frac{d}{dx}\left[\varepsilon_\mathrm{sur}(\rho_\mathrm{v} v_\mathrm{v} s_\mathrm{v} h_\mathrm{v} + \rho_l v_l s_l h_l) - \lambda\frac{dT}{dx}\right] = r(x). \tag{23}$$

Assuming that the additive law of the summation of heat conductivities holds for λ, we have

$$\lambda = \lambda_l s_l \varepsilon_{\text{sur}} + \lambda_v s_v \varepsilon_{\text{sur}} + \lambda_{\text{sol}}(1 - \varepsilon_{\text{sol}}). \tag{24}$$

The following boundary conditions must be added to Eqs. (20)-(23):

$$p_{x=0} = p_1;\ p_{x=l} = p_2;\ \lambda \frac{dT}{dx}\Big|_{x=0} = [\rho_l v_l c_{pl}(T - T_\infty)]_{x=0};\ -\lambda \frac{dT}{dx}\Big|_{x=l} = q. \tag{25}$$

The above system of transport equations holds only within the framework of our model of liquid bubbling in a porous solid.

NOTATION

x	is the coordinate;
t	is the time;
ρ_l	is the density;
ρ_v	is the vapor density;
η_l	is the dynamic viscosity of the liquid;
η_v	is the dynamic viscosity of vapor;
λ_l	is the thermal conductivity of the liquid;
λ_v	is the thermal conductivity of vapor;
λ_{sol}	is the thermal conductivity of the porous matrix;
ε_{sur}	is the surface porosity;
p	is the pressure;
T	is the temperature;
$p_s(T)$	is the saturated vapor pressure at the temperature T;
μ	is the molecular weight;
R^*	is the universal gas constant;
h_v	is the enthalpy of vapor;
h_l	is the enthalpy of the liquid;
c_{pl}	is the specific heat of the liquid at constant pressure;
l	is the length of porous specimen;
q	is the incident thermal flux;
$r(x)$	is the density of external energy sources;
T_∞	is the liquid temperature at a point remote from the porous wall;
p_1	is the inlet pressure;
p_2	is the outlet pressure;
k	is the permeability of the porous structure.

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